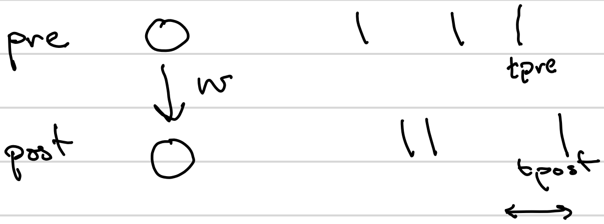
How does a weight matrix arise?

- Spike Time Dependent Plasticity (Gerstner 19.2.2)



$\Delta t = t_{post} - t_{pre}$. ($>0$ if tpre before t_{post})

- Bi & Poo J. Nsci 1999.

STDP KERNEL:
$$\Delta W(\Delta t) = \begin{cases} A_+ e(-|\Delta t|/\tau_+) > 0, & \Delta t > 0 \quad \text{LTP} \\ A_- e(-|\Delta t|/\tau_-) < 0, & \Delta t < 0 \quad \text{LTD} \end{cases}$$

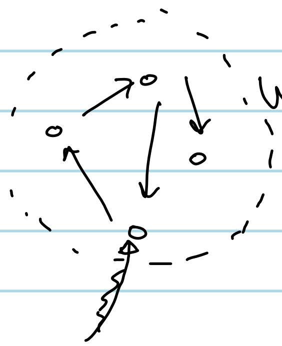
Let $C(\Delta t)\,dt = P(\Delta t \in [\Delta t, \Delta t + dt])$.

"Cross-Covariance density" (or... Cross-Correlation function).

THEN: $$\langle \Delta W \rangle = \int d(\Delta t)\; C(\Delta t)\, \Delta W(\Delta t)$$

$$\sim \frac{d}{dt} W$$, over long timescales

• Self-Consistent Models of Plasticity in Recurrent Spiking Nets •



W_{ij}, coupling among N neurons

BACKGROUND: μ_i, σ_i^2, from unmodeled neurons

$$y_i(t) = \sum_k \delta(t - t_k^i) \quad \text{SPIKES}$$

SPIKE RATE:

Linear response filter, from diffusion approx.

$$r_i(t) = r_{0,i} + \sum_j W_{ij} \cdot G_i(t) * y_j(t)$$

$r_{0,i}$: Determined by μ, σ^2

Let $\tilde{y}_i(\omega) = F(y_i(t)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-2\pi i \omega t} y_i(t)\, dt$

(Wiener-Khinchine)

FACT:

Let $\underbrace{C_{ij}(\omega)}_{\text{for neur. } i,j} = F(c_{ij}(\Delta t))$. Then $C_{ij}(\omega) = \mathbb{E}\left(\tilde{y}_i^*(\omega)\, \tilde{y}_j(\omega)\right)$

Wiener-Hopf Transformation (Hawkes 1972)... Equivalent to following formal manipulation... yields $c_{ij}(\omega)$

$$\tilde{y}_i(\omega) = \underbrace{\tilde{y}_{i,0}(\omega)}_{\text{Unperturbed spike train}} + \sum_j W_{ij}\, \tilde{G}_i(\omega)\, \tilde{y}_j(\omega)$$

$$\vec{y}(\omega) = \vec{y}_0(\omega) + G(\omega)\, W \vec{y} \quad ; \quad \text{let } \overline{W} = G\, W(\omega)$$

$G(\omega)$: diagonal Matrix

$$(I - \overline{W})\, \vec{y}(\omega) = \vec{y}_0(\omega)$$

$$\vec{y}(\omega) = (I - \overline{W}(\omega))^{-1}\, \vec{y}_0(\omega)$$

$$C(\omega) = \mathbb{E}\left(\vec{y}^{\,*}(\omega)\ \vec{y}^{\,T}(\omega) \right)$$

$$= \left(I - \bar{W}^{*}(\omega)\right)^{-1} \underbrace{\mathbb{E}\left(y_0^{*}(\omega)\ y_0^{T}(\omega) \right)}_{C_0(\omega)} \left(I - \bar{W}(\omega)\right)^{-T}$$

$$= A(W; \omega) \quad , \text{ simple matrix function}$$

Inverse F.T. $\longrightarrow$ $C(\Delta t) = \underbrace{\mathcal{F}^{-1}}_{\text{inverse F.T.}} \left(A(W;\omega) \right)(\Delta t)$

<u>THUS:</u>

$$\frac{d}{dt} W = \int d(\Delta t)\ \underbrace{\Delta W(\Delta t)}_{\text{STDP Kernel}}\ \mathcal{F}^{-1}\left(A(W;\omega) \right)(\Delta t)$$

$$\frac{d}{dt} W = B(W)$$

$$\boxed{\text{Closed form, explicit ODE for } W!}$$

- Ocker + Doiron, Plos CB 2015 (Slide)